# METHODS TO EVALUATE THE DYNAMIC STABILITY OF STRUCTURES – SHAKE TABLE TESTS AND NONLINEAR DYNAMIC ANALYSES

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## ABSTRACT

This paper aims to understand the phenomenon of dynamic instability in structures better and to evaluate and suggest methods to predict collapse limit states of structures during earthquakes, based on findings of recent shake table tests and nonlinear dynamic analyses conducted at Stanford University. Simple models that collapsed due to the story mechanism were used as test specimens. Data from nineteen experiments suggests that current methods of nonlinear dynamic analysis (using the OpenSees program in this case) are very accurate and reliable for predicting collapse and tracing the path of the structure down to the ground during collapse. Moreover, it is found from the experiments that for non-degrading structures, a static pushover analysis-based estimate of collapse drift can be successfully applied to predict the dynamic collapse or instability due to P-Δ effects. The rationale for this is that the structure has a very elongated period at the point of global instability, virtually insulating it from the ground motion and justifying the use of a static analysis-based drift. Finally, the paper directs the readers to a valuable database of test data from collapse tests of a "clean" structure, which can be used for further verification studies.

## INTRODUCTION

In theory, the current state of nonlinear dynamic structural analysis has the ability to evaluate the performance of structures till collapse occurs, ideally tracing the path of the structure down to the ground. However, performing shake table tests of a structure till collapse can be dangerous and often expensive, so this ability of the analysis methods has not been extensively verified. Also, since test data for such collapse situations is sparse, understanding of such behavior has not been adequately studied. An important problem is to understand the concept of dynamic collapse and to relate this phenomenon to simpler design guidelines and seismic demands, such as a critical interstory drift, which may be calculated using a simpler method, such as a pushover analysis. Such a method, which relates the statically calculated drift to the dynamic collapse, could prove extremely important especially when it is unfeasible to carry out detailed nonlinear dynamic analysis.

The experimental and analytical study at Stanford was initiated with these problems in mind. The important aim was to answer these questions based on actual collapse test data, which is rare to find. In the process a well-documented database of the all the test results was created

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which can be used by other researchers for verification of their own analytical studies. This complements a similar investigation by Vian et al [5]. Small-scale structures with story height ten inches and bay width two feet were used as the test specimens. All of these structures were designed to fail in the story mechanism with highly localized plastic hinging of the columns at the top and bottom ends, due to the lateral loads as well as the P-Δ effect from the gravity loads. The structures were designed and the ground motions were scaled to different levels to push some structures to the brink of collapse while collapsing some others. This tested the abilities of the analysis to predict response under very large displacements, and also provided valuable data from collapsed structures.

The paper begins by briefly describing the test setup and plan, follows with an overview of the analysis, and concludes with observations about the accuracy of the analysis by suggesting a pushover-based methodology for predicting collapse in non-degrading structures.

## TEST PLAN AND SETUP

A total of nineteen tests were conducted as part of this study. The basic structural configuration was in the form of four flat columns and a steel mass on top, which also served as a rigid diaphragm. The columns were cut from 1018 carbon steel and had a cross-section of 1/8” by 1”. The mass weighed 320 lbs, and was in the form of steel plates. The clear height of the columns in the model was 10 inches. The orientation of the columns was so that the weak axes were perpendicular to the direction of motion. The structure measured 12” by 24” in plan, the longer dimension aligned in the direction of motion. This created a high out-of-plane stiffness against torsional effects. The columns were connected to the base plate and the mass be means of a clamping mechanism, which allowed convenient replacement of the columns for different tests. This design was chosen to ensure a clean, unidirectional behavior of the structure, free from complicated behavior. The structure fails in a story mechanism, with columns bending in double-curvature, with plastic-hinges at the top as well as the bottom of the columns. Fig. 1 shows a photo of the specimen.

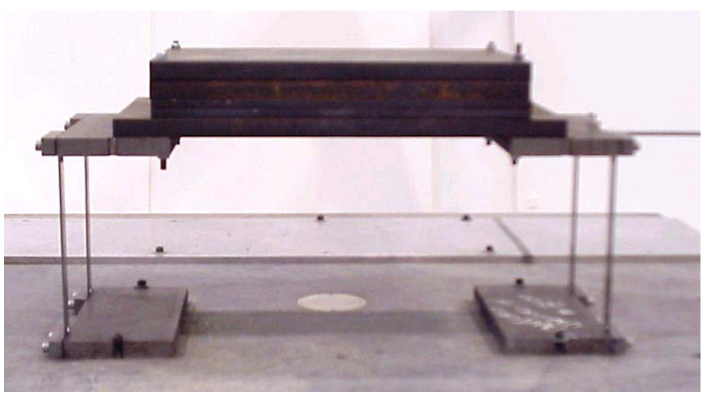


FIGURE 1. Specimen Configuration

The basic structure shown above will be referred to as structure A. The natural period of vibration of specimens with this configuration was found to be between 0.42 and 0.45 seconds from free vibration tests, while the damping was found to be around 1 percent of critical. The strength of this structure, expressed as in terms of the yield base shear to weight was $V_y/W$ = 1.03. This indicates that the structure was very strong, and would not fail even if it was standing on its side. To reduce the strength of the structure, ½ inch diameter holes were drilled at the plastic hinge locations in the columns. This reduced the strength ratio of the structure $V_y/W$ to 0.6, as compared to 1.03 of structure A. This weakened structure will be referred to as structure B. The period for this structure was slightly elongated (in the 0.48-0.5 second range), whereas the damping was roughly equal to that of structure A. The stability coefficient $\theta$ was 0.17 for each of the structures, which is comparable to realistic structures.

Data was acquired for many quantities, the important ones being the acceleration and displacement of the table, as well as the acceleration and displacement of the specimen. The specimen acceleration was measured at eccentric locations to monitor any torsional behavior that the model may experience.

Two ground motions were used for testing the specimens. The choice of these was governed not only by the structural period and properties, but also by the limitations of the shake table that needed to achieve certain levels of acceleration within specified boundaries of displacement. The two ground motions used were (Fig. 2 shows the acceleration response spectra of the two records used) –

1. Northridge at Obregon Park, Los Angeles – This is a regular earthquake (far-field type record), with a spectral acceleration at the initial period being around 1g. We will refer to this record as OBR.
2. Northridge at Pacoima Dam – This earthquake is recorded on the crest of the Pacoima Dam, which causes it to much more intense, the spectral acceleration at the initial period being about 3g. This will be referred to as PAC.

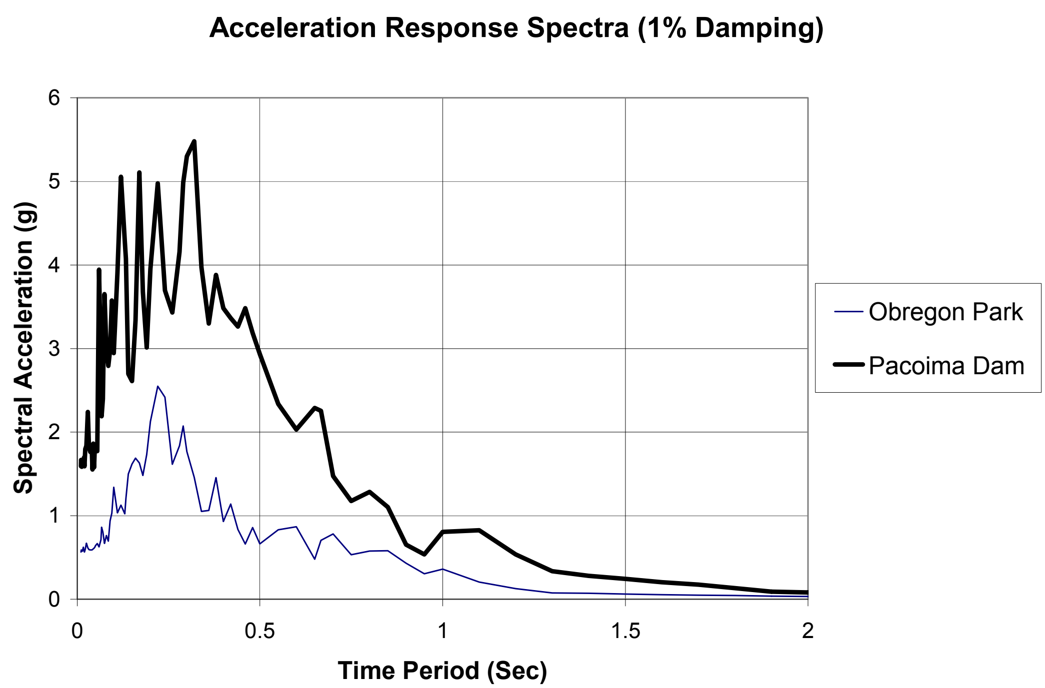



FIGURE 2. Acceleration Response Spectra of the earthquakes used in the investigation

The testing program was divided into three smaller series, which involved varying the structural configuration as well as the earthquakes. The series were –

Series 1 – Structure A was used along with the OBR record. Since this was a strong structure, the structure did not collapse dramatically, but sustained very severe (40-50%) drift. It remained standing after each of the tests.

Series 2 – Structure B was used along with the OBR record. This structure was weak, and hence collapsed dramatically during the applied earthquake.

Series 3 – Structure A was used with the PAC record. The PAC record hit the structure much harder and much more severe drift were recorded (of the order of 80%), as compared to Series 1.

Within each of the series, the records were run multiple times, typically a low-level test was conducted to verify the elastic response of the structure, and this was followed by the high level test. The earthquakes were scaled such that to produce different levels of drift. Table 1 lists the tests that were run, and provides numbers for the tests, which we will refer to in subsequent sections.

TABLE 1. Test Matrix for the experimental program

| Test Number | Earthquake Record | Structure Type | $S_a(T_1)$ in g |
|---|---|---|---|
| *Series 1* | | | |
| 1 | OBR | Structure A | 2.88 |
| 2 – 4 | OBR | Structure A | 3.36 |
| 5 – 10 | OBR | Structure A | 3.84 |
| *Series 2* | | | |
| 11 –13 | OBR | Structure B | 1.92 |
| 14 – 16 | OBR | Structure B | 1.44 |
| *Series 3* | | | |
| 17 – 19 | PAC | Structure A | 5.3 |

Running the tests at different intensity levels within the same series (the same earthquake and structure) helped us evaluate the range over which the analysis models were effective, and beyond what levels of drift angle we need to be careful about trusting the analysis prediction. Simply running one series at one intensity level would not have given us this appreciation of the effectiveness of the analysis.

## ANALYSIS FEATURES AND CALIBRATION

The computer analysis models were carefully designed and calibrated to ensure the best possible prediction of response. The analysis was carried out on the open-source software OpenSees, being developed at the Pacific Earthquake Engineering Center (PEER) in Berkeley, California. The key features of the model, and the assumptions involved are outlined in this section –

1. Two dimensional Nonlinear Time History Analysis
2. Concentrated Mass at ends of top beam
3. Elastic Columns and rigid beam
4. Inelastic SDOF zero length rotational spring at plastic hinge locations at ends of columns (See Fig. 3)
5. Large displacement analysis (Corotational formulation in OpenSees)

6. Small deformation geometric nonlinear analysis (no element bowing/arching included)

Fig.3 shows a schematic of the analysis model used in OpenSees.

The single degree of freedom rotational springs were calibrated from monotonic as well as cyclic static bending tests of the column flats in a standard load frame. Special fixtures were designed to mimic the real connection conditions. These tests were conducted for the columns for structure A as well as B, i.e., with and without drilled holes in them.

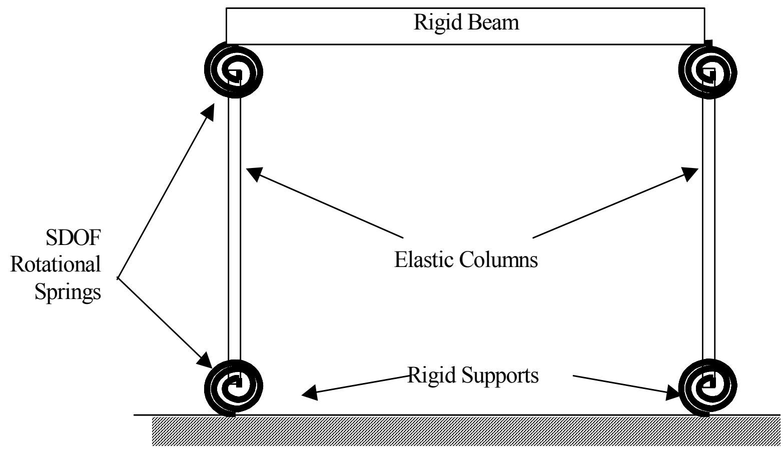


FIGURE 3. Schematic of Computer Model of the Structure

The model chosen in this case for modeling the SDOF spring hysteretic response was the Giufré-Menegotto-Pinto plasticity model. The model has a yield envelope and a nonlinear hardening exponential law. The yield envelope is defined using a yield point and hardening modulus, and the curve is interpolated between the initial and the final slopes through a shape or curvature parameter R. See Fig. 4, and Eq. 1. Table 1 lists values used for each of the parameters used for the model.

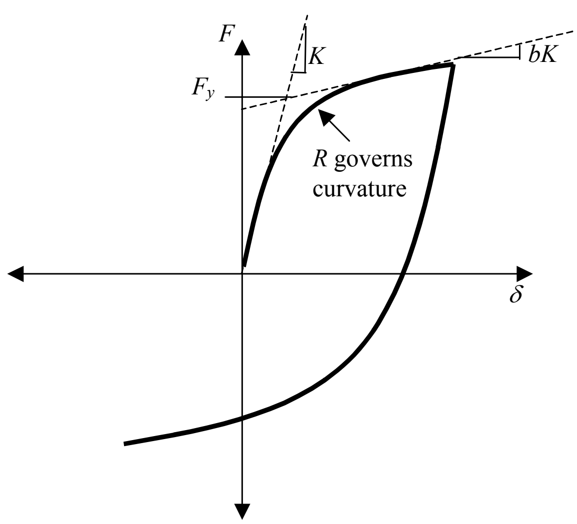


FIGURE 4. Plasticity Model for SDOF Springs

$$F = K\delta\left(b + \frac{1-b}{\left(1+\left(K\delta / F_y\right)^R\right)^{1/R}}\right) \quad (1)$$

$F$ = Force
$\delta$ = Displacement
$K$ = Initial Stiffness
$F_y$ = Yield Force
$b$ = Hardening Coefficient
$R$ = Curvature Exponent

This Material Model is available as the Steel02 Model in the OpenSees program. The model is calibrated so as to account for the nonlinear hysteretic behavior of the plastic hinge as well as the elastic flexibility of the beam-column connections. The values used for the various parameters in this model are summarized in Table 2. It should be noted here that the Steel02 model is used to model a single degree of freedom rotational spring, so the Force term F is actually the moment in the spring, whereas the displacement term δ is the rotational deformation of the spring.

TABLE 2. Model Parameters for the Plasticity Model used

| Parameter | Value | Units |
|---|---|---|
| $K$ | 8.0 | Kip-in/Radian |
| $F_y$ | 0.4125 for structure A, 0.2398 for structure B | Kip-in |
| $\beta$ | 0.02 | None |
| $R$ | 16.8 | None |

## THE USEFULNESS OF NONLINEAR DYNAMIC ANALYSIS TO PREDICT COLLAPSE

Time history plots of lateral displacement from the experiments were compared to the time-history plots obtained from the analytical simulations of the same experiment, and the simulations were found to predict the analytical predictions with reasonable accuracy. It should be noted here that the shake table acceleration data was used to run the analysis, as opposed to the original earthquake, to compensate for any inconsistencies in table behavior. A mean square relative error calculation scheme was used to quantify this accuracy. The formula used for calculating the relative error is shown in Eq.2. –

$$e_{relative} = \frac{\sqrt{\dfrac{\sum_{i=1}^{n}(\Delta_{test}^{i} - \Delta_{analysis}^{i})^2}{n}}}{\Delta_{test}^{\max}} \tag{2}$$

Where,

$\Delta_{test}^{i}$ = The displacement measured at the i$^{th}$ time instant from the test

$\Delta_{analysis}^{i}$ = The displacement calculated at the i$^{th}$ time instant from analysis

$\Delta_{test}^{\max}$ = The maximum displacement observed in the test, to normalize the mean - squared error calculated in the numerator of the expression

Such a measure is a very stringent measure of the relative error, since it measures cumulative error over the entire length of the record, and hence includes the effects of constant errors like offsets, which may be due to slip (in the joints) in the actual structure, which is not picked up by the analysis. Table 2 lists the relative errors calculated for each of the tests.

On an average, the relative error is about 15% of the maximum drift, which is a very encouraging number. This demonstrates the usefulness and accuracy of the time history analysis. Fig. 5 shows the time history graphs from the test and analysis for Test #3, for the purpose of illustration. A visual examination of the graph shows excellent agreement. The relative error measured for this experiment is 20%, which helps put the accuracy of the other predictive analyses in perspective.

TABLE 2. Relative Errors from the Test Data

| Test Number | Relative Error | Test Number | Relative Error |
|---|---|---|---|
| 1 | 19% | 11 | 10% |
| 2 | 24% | 12 | 6% |
| 3 | 20% | 13 | 11% |
| 4 | 32% | 14 | 6.6% |
| 5 | 26% | 15 | 34% |
| 6 | 16% | 16 | 15% |
| 7 | 20% | 17 | 2% |
| 8 | 26% | 18 | 3% |
| 9 | 10% | 19 | 7% |
| 10 | 25% | | |

Though the relative error is a useful estimate of the accuracy of analysis, it is often important to think about the usefulness of the analysis in predicting certain important demand parameters, such as maximum drift or residual drift. For this purpose, incremental dynamic analyses (IDA) curves are generated, which indicate the maximum or residual drift in the structure as a function of the earthquake intensity measure, which typically is the spectral acceleration at the initial time period of the structure. The tests results, which have been run at specific values of the intensity measure, are then superimposed on the IDA curve to make a comparison between the response predicted by the analysis and that observed during the tests. Figs. 6 through 8 show the IDA curves for the three different configurations tested (Series 1,2 and 3), for the residual as well as maximum drift. The data points observed in the corresponding tests are superimposed for a comparison.

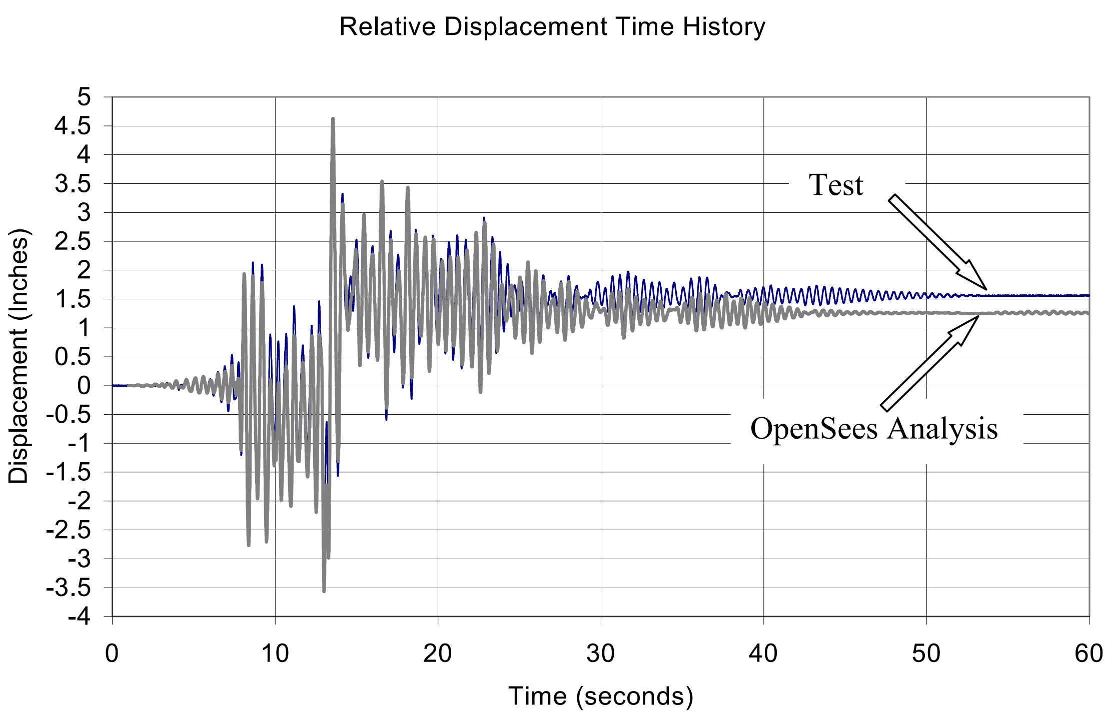



FIGURE 5. Comparison of Test Results and Analytical Predictions for Test #3 (20% Relative Error)

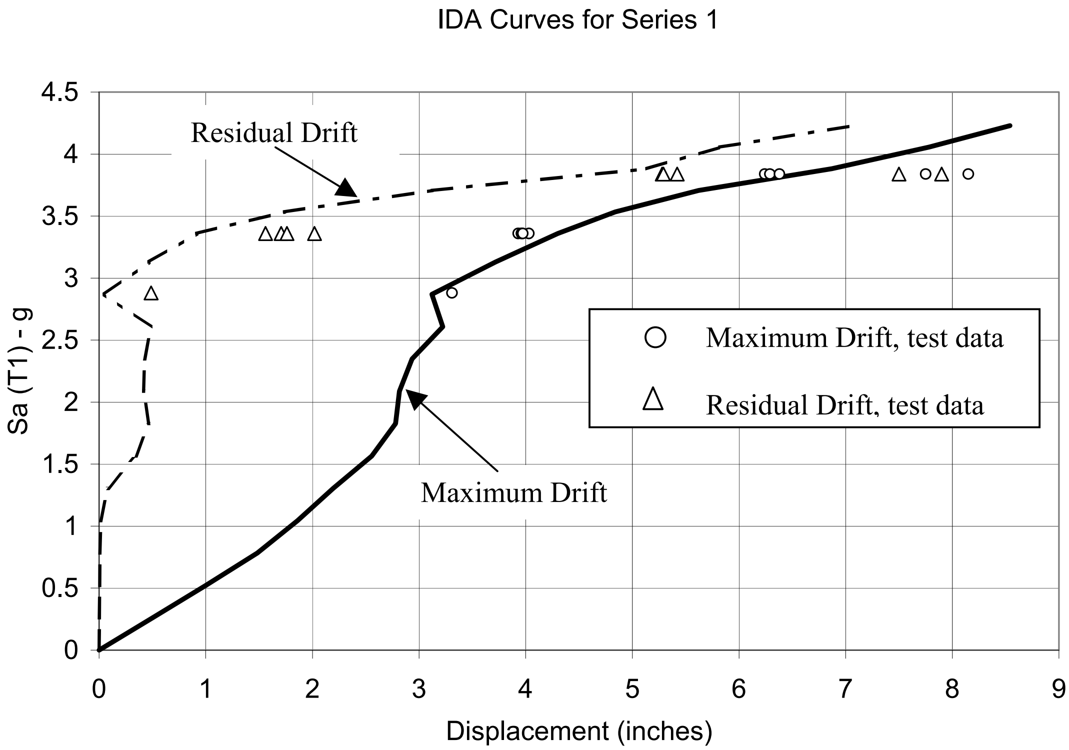



FIGURE 6. Comparison of the IDA Curves and Tests for Series 1

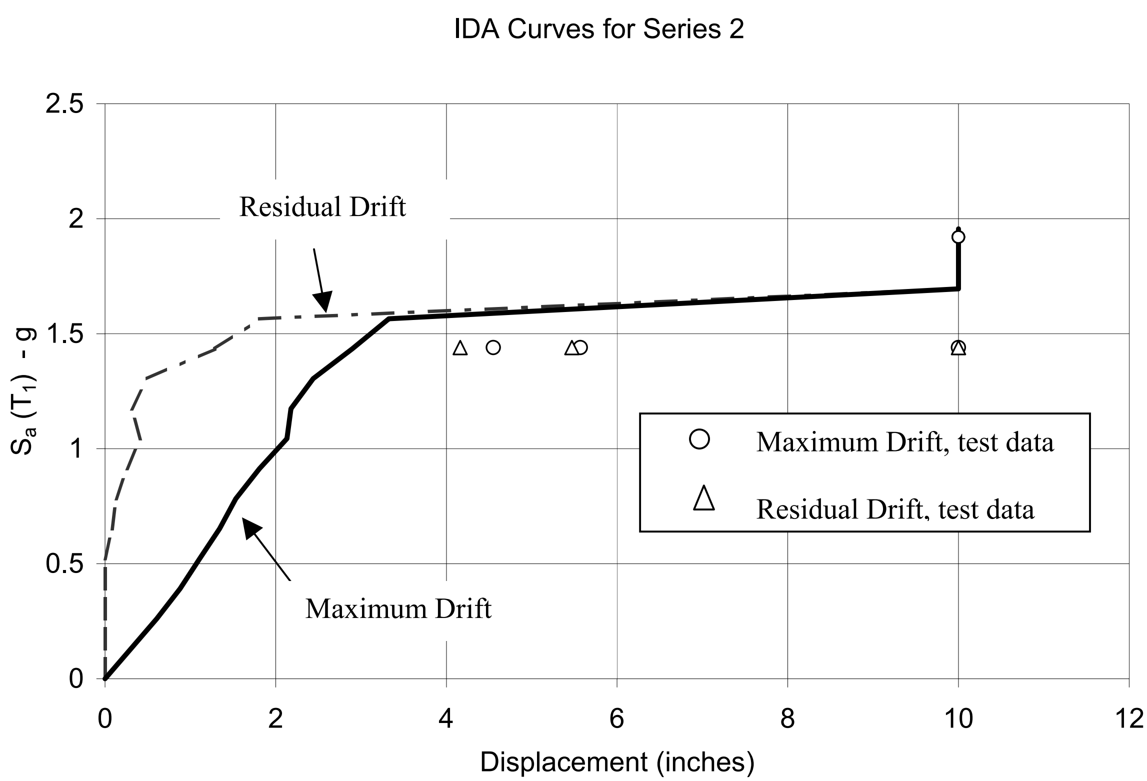



FIGURE 7. Comparison of the IDA Curves and Tests for Series 2

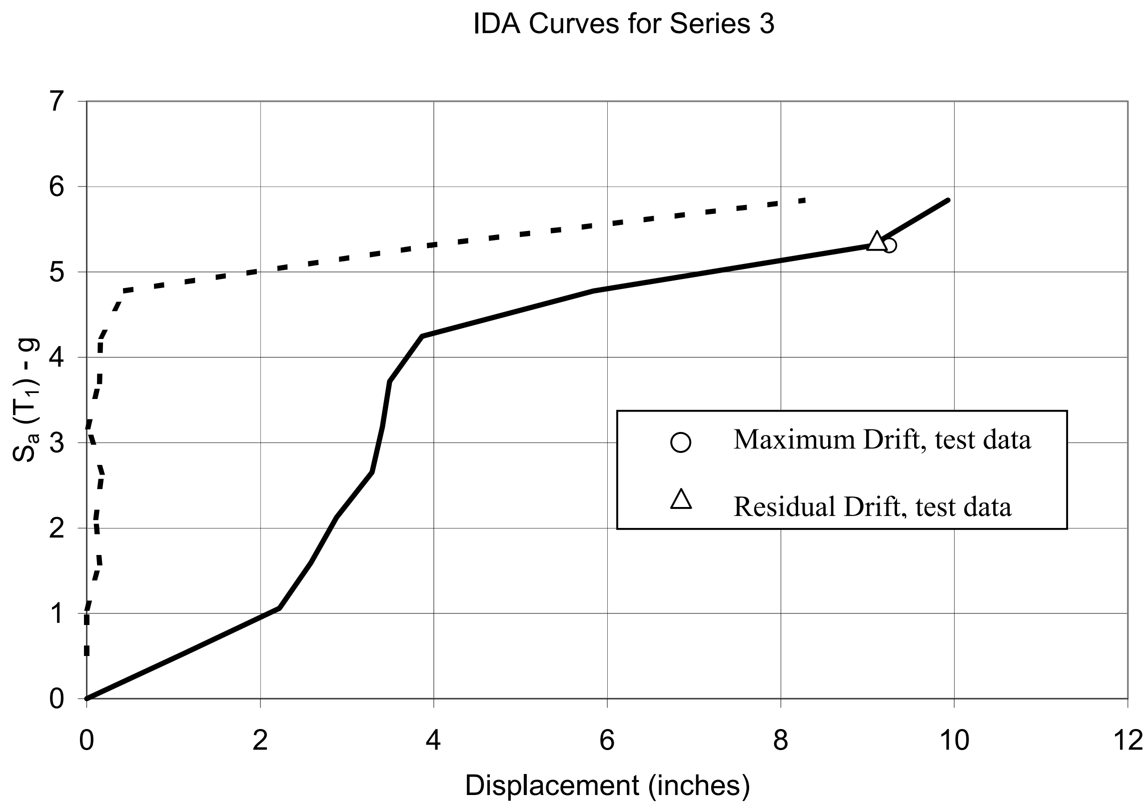


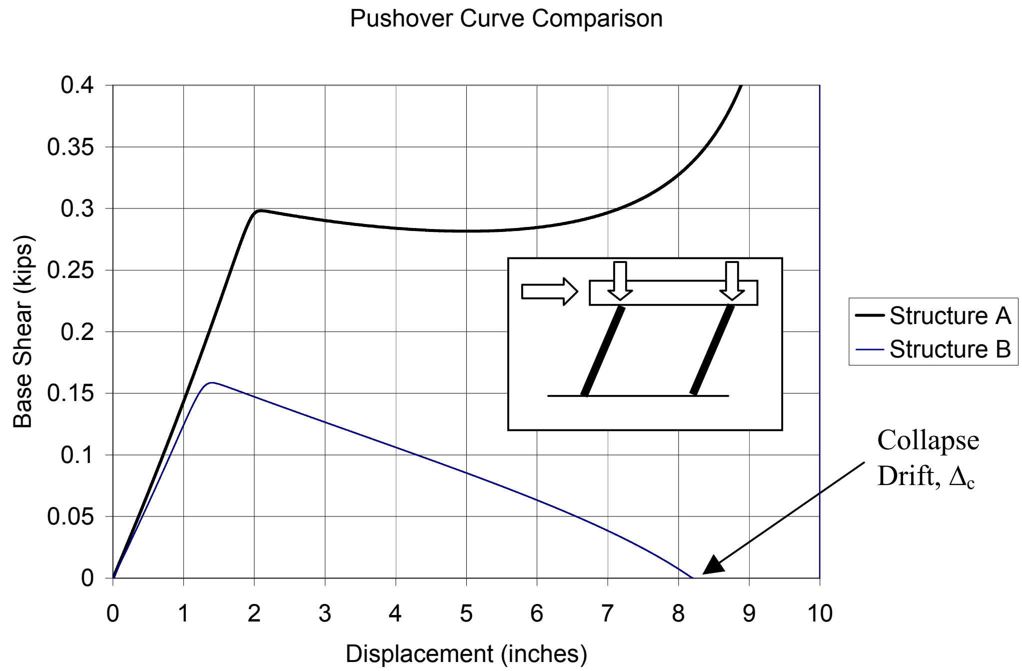

FIGURE 8. Comparison of the IDA Curves and Tests for Series 3

Looking at the IDA curves, we can infer that the analysis does a reasonably accurate job of predicting the maximum and residual drift in the structures. Moreover, we observe that the scatter in the experimental results increases as the structure nears collapse (this can be seen clearly in Figs 6 and 7), where the scatter in the test results is higher at the stronger earthquake levels. Once the structure collapses, of course, the scatter is zero, since the displacement is equal to story height.

## METHODOLOGY FOR PREDICTING DYNAMIC INSTABILITY

Based on the test data and the analysis results, we can propose that static pushover analysis of the structure can give us reasonable insight into the dynamic instability behavior, by using test results for relating the static instability drift to a limit drift for dynamic conditions. Static pushover analyses were run for both structure type A and B. The pushover curves (base shear versus lateral displacement) are shown in Fig. 9.

The pushover curve for structure A actually rises as the drift angle increases. This is because of large displacements in the columns that cause them to behave like axial force members. This pushover curve predicts that the structure will never become unstable statically; rather it will continue to pick up force at larger and larger displacements. The pushover curve for structure B, which is much weaker, presents a different picture. The structure yields at roughly 60% of the force level as structure A, but then the P-Δ effects take over, causing the structure to gradually lose its base shear capacity. At about 8.2 inches of displacement, the base shear in the structure is zero, which means that the P-Δ effect alone can drive the structure to the ground. This can be referred to as the “collapse drift” of the structure. We will denote it by $\Delta_c$. Beyond this point, the force becomes negative, which means that the structure actually needs to be held up by the horizontal force.




Having established these quantities, we can shift our attention to the dynamic tests. Tests in Series 2 (# 11 through #16) are directly relevant, since they employ Structure type B. It is observed that in any of these tests, the structure collapses if the drift at any time exceeds $\Delta_c$. This collapse is sudden, immediate and on the same excursion when the drift exceeds this value. Tests 11,12,13,16 show this.

On the other hand, when the drift does not exceed $\Delta_c$ at any time, the structure survives the earthquake and remains standing, though in a damaged state. This is observed in Tests 14 and 15. One is tempted to ask the question if this is more than just coincidence. Though there are differences in the characteristics of different ground motions, and differences between cyclic and monotonic load histories, one can assume that near the point of collapse, the structure has an extremely elongated period, since the equivalent stiffness at this point is almost zero. This would mean that the ground motion would not be able to exercise appreciable influence on the structure. The structure at this point is governed almost totally by P-Δ effects and has very little influence of the horizontal motion of the structural mass. Moreover, if we have a practically non-degrading structure (like the one actually tested), the strength of the structure as derived from monotonic and cyclic analyses will not be appreciably different.

Test series 1 and 2 never show collapse in any of the specimens. This is consistent with the idea of using the statically determined $\Delta_c$, which, for these series (Structure A) is infinity. This means that the P-Δ effect will never be large enough to drive the structure to collapse all by itself. Based on these facts, we can say that there definitely is a relationship between the $\Delta_c$ and the dynamic point of instability.

However, the exact relationship between the statically estimated collapse drift, and the dynamic instability point can be made clear only after more tests or reliable analyses. This would remove sources of uncertainty and scatter that may arise due to the different characteristics of ground motions.

It can be visualized that further studies of this type could use more data to relate the dynamic collapse drift to the static collapse drift in a more precise way, something like $\Delta_D = F. \Delta_c$, where F could to consider the uncertainty in the process, $\Delta_c$ being a nominal value. This would require more testing or analysis geared specifically to this end.

## CONCLUSIONS

Many interesting observations and conclusions were made from the testing and analysis program. Some of them relate to the accuracy of analysis, whereas others relate to the structural behavior, and the different response of the structures to the ground motions. To summarize –

1. Excellent agreement was obtained between test results and time-history analysis. Two different approaches are used to evaluate the accuracy of the analyses. First, a root mean squared relative error between the test and the analysis is reported for each of the tests, and this is found to be around 15% on an average This, along with a visual inspection of the time-history graphs (tests superimposed on analysis) confirms the agreement of the tests and the analysis. The IDA curves for the structure show good agreement with the predicted maximum drift and residual drift.

2. A static pushover analysis of the structure is used to better understand some of the results observed in the shake table testing. The collapse drift for the static pushover (for structure B) was found to be 8.2 inches. It was observed that all structures that were subjected to drifts larger than this value during the shaking eventually collapsed, whereas all those that were not, survived the earthquake. For structure A, this drift was larger than the length of the column, and consequently, none of the structures were seen to collapse. This is an important observation, since it relates the statically observed collapse drift ratio from (P-Δ) analysis to real dynamic collapse, i.e., for earthquake design, using this drift to calculate the R factor of a structure might be a reasonable approach. A possible explanation for this behavior is the independence of structural response to ground motion in the region of collapse, due to the extremely elongated period of the structure. An important feature to note here is the non-degrading property of the structure that enables us to relate the pushover to the dynamic collapse. Further studies, with structures of different strength, and analyses with different ground motions can contribute to this approach of estimating dynamic stability based on static instability indices.

3. The scatter is structural response is greater near collapse, than it is at lower-intensity earthquakes. This leads one to believe that collapse of structures might be more sensitive to imperfections in construction than other limit states.

4. A database of test data has been generated at Stanford University, and more tests are planned in the future. This database provides a valuable source of data for calibration and verification of other material or collapse models. This complements the work done by Vian et al, at MCEER, SUNY Buffalo.

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